

# Energy Extraction from a Rotating Black Hole by Magnetic Reconnection in Ergosphere

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## ABSTRACT

We investigate mechanisms of energy extraction from a rotating black hole in terms of negative energy-at-infinity. In addition to the Penrose process through particle fission, the Blandford-Znajek mechanism by magnetic tension, and the magnetohydrodynamic Penrose process, we examine energy extraction from a black hole caused by magnetic reconnection in the ergosphere. The reconnection redistributes the angular momentum efficiently to yield the negative energy-at-infinity. We derive a condition for the process to operate in a simple situation, where the plasma is incompressible and the magnetic energy is converted completely to the plasma kinetic energy locally. Astrophysical situations of magnetic reconnection around the black holes are also discussed.

*Subject headings:* Black hole physics, magnetohydrodynamics: MHD, relativity, methods: analytical, galaxies: nuclei, gamma rays: bursts, plasmas

## 1. Introduction

Energy extraction from a rotating black hole interests us not only as engines of relativistic jets from active galactic nuclei (AGNs), micro-quasars ( $\mu$ QSOs), and gamma-ray bursts (GRBs) (Meier et al. 2001), but also as fundamentals of black hole physics. The horizon of the black hole is defined as the surface where no matter, energy, and information pass through outwardly. On the other hand, reducible energy from the rotating black hole is given by

$$E_{\text{rot}} = \left[ 1 - \sqrt{\frac{1}{2} \left( 1 + \sqrt{1 - a^2} \right)} \right] Mc^2, \quad (1)$$

where  $M$  is the mass,  $a$  is the rotation parameter of the black hole, and  $c$  is the speed of light (Misner et al. 1970). It corresponds to the rotational energy that can be extracted principally.

Several kinds of distinct mechanisms have been proposed for the extraction of the rotational energy: e.g. Penrose process, super-radiant scattering, Blandford-Znajek mechanism, magnetohydrodynamic (MHD) Penrose process, and modified Hawking process (Penrose 1969; Press et al. 1972; Ruffini et al. 1975; Blandford et al. 1977; Hirotani et al. 1992; van Putten 2000). Here we mention three kinds of mechanism of the black hole energy extraction among them: the Penrose process, the Blandford-Znajek mechanism, and the MHD Penrose process, while the super-radiant scattering and the modified Hawking process may be related with the high energy phenomena such as origins of gamma-ray bursts and the ultra-high energy cosmic rays (van Putten 1999; van Putten 2000; Pierre Auger collaboration 2007). The Penrose process involves production of particles with negative energy-at-infinity via strong fission or particle interaction in the ergosphere (Penrose 1969). It needs drastic redistribution of angular momentum to produce the negative energy-at-infinity. Although this process clearly shows a possibility of energy extraction from the black hole, it is improbable as engines of astrophysical jets, because of poor collimation of particles and poor event rate. That is, the Penrose process accelerates the particles toward the equatorial plane, not toward the axis direction, and needs the azimuthal relativistic fission at not so wide region in the ergosphere. Blandford & Znajek (1977) proposed a mechanism of energy extraction from a rotating black hole through force-free magnetic field. Their analytic steady-state solutions show the direct energy radiation from the horizon, which looks contradictory to the definition of the black hole horizon (Punsly et al. 1990). As we discuss in the next section, this mechanism also utilizes the negative energy-at-infinity. In this case, however, the negative energy-at-infinity is sustained not by the particle or matter, but by the electromagnetic field. Angular momentum of the electromagnetic field is redistributed by the magnetic tension through almost mass-less plasma to produce the negative energy-at-infinity of the field. The magnetic tension may also redistribute the angular momentum of the plasma to yield the negative energy-at-infinity of the plasma, when the plasma has non-zero mass density (Hirotani et al. 1992). It is called the MHD Penrose process. This energy extraction was confirmed by the numerical simulations based on the general relativistic magnetohydrodynamics (GRMHD) (Koide et al. 2002; Koide 2003).

It is noted that magnetic reconnection also redistributes angular momentum of the plasma to form the negative energy-at-infinity because it produces a pair of fast outflows with the opposite directions from the reconnection region. Then the rotational energy of the black hole can be extracted through the induced negative energy-at-infinity of the plasma. In the present paper, we derive a condition for the process to operate in a simple situation

for the incompressible plasma, where all the magnetic energy is converted to the plasma kinetic energy.

In §2, we review the mechanisms of energy extraction from the rotating black hole in terms of the negative energy-at-infinity. In §3, we examine the operation condition of the energy extraction from the black hole induced by magnetic reconnection in the ergosphere using a simple model. In §4, we discuss astrophysical situations where the magnetic reconnection happens in the ergosphere.

## 2. Penrose process, Blandford-Znajek mechanism, and MHD Penrose process

We use the Boyer-Lindquist coordinates  $(ct, r, \theta, \phi)$  to describe the space-time around a rotating black hole. The line element of the axisymmetric, stationary space-time around the black hole is written by

$$ds^2 = -\alpha^2(ct)^2 + \sum_{i=1}^3 h_i^2 (dx^i - \omega_i dt)^2 = -\alpha^2(ct)^2 + \sum_{i=1}^3 (h_i dx^i - c\alpha\beta_i dt)^2, \quad (2)$$

where  $h_i$  is the scale factor of the coordinate  $x^i$ ,  $\omega_i$  is the angular velocity describing a frame-dragging effect,  $\alpha$  is the lapse function, and  $\beta_i = h_i \omega_i / (c\alpha)$  is the shift vector. For the Kerr metric (Misner et al. 1970), we have

$$\alpha = \sqrt{\frac{\Delta\Sigma}{A}}, \quad h_1 = \sqrt{\frac{\Sigma}{\Delta}}, \quad h_2 = \sqrt{\Sigma}, \quad h_3 = \sqrt{\frac{A}{\Sigma}} \sin\theta, \quad \omega_1 = \omega_2 = 0, \quad \omega_3 = \frac{2cr_g^2 ar}{A}, \quad (3)$$

where  $\Delta = r^2 - 2r_g r + (ar_g)^2$ ,  $\Sigma = r^2 + (ar_g)^2 \cos^2\theta$ ,  $A = \{r^2 + (ar_g)^2\}^2 - \Delta(ar_g)^2 \sin^2\theta$ , and  $r_g = GM/c^2$  is the gravitational radius of the black hole.

This metric has translational symmetry with respect to  $t$  and  $\phi$ , so that we obtain the conservation law

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} (\sqrt{-g} T^{\mu\nu} \xi_\nu) = 0, \quad (4)$$

where  $g = \text{Det}(g_{\mu\nu}) = -(\alpha h_1 h_2 h_3)^2$  is the determinant of the metric tensor,  $T^{\mu\nu}$  is the energy-momentum tensor, and  $\xi^\nu$  is the Killing vector. When we adopt one component approximation of the plasma, we have

$$T^{\mu\nu} = pg^{\mu\nu} + \mathfrak{h}U^\mu U^\nu + F_\sigma^\mu F^{\sigma\nu} - \frac{1}{4}g^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}, \quad (5)$$

where  $p$  is the proper pressure,  $\mathfrak{h} = e_{\text{int}} + p$  is the enthalpy density,  $U^\mu$  is the four-velocity, and  $F_{\mu\nu}$  is the electromagnetic field-strength tensor (Koide et al. 1999). The thermal energy

density is given by  $e_{\text{int}} = p/(\Gamma - 1) + \rho c^2$  for adiabatic plasma, where  $\Gamma$  is the adiabatic index and  $\rho$  is the proper mass density.

When we consider the Killing vectors  $\xi^\nu = (-1, 0, 0, 0)$  and  $(0, 0, 0, 1)$ , we get the energy and angular momentum conservation laws,

$$\frac{\partial e^\infty}{\partial t} = -\frac{1}{h_1 h_2 h_3} \sum_i \frac{\partial}{\partial x^i} (h_1 h_2 h_3 S^i), \quad (6)$$

$$\frac{\partial l}{\partial t} = -\frac{1}{h_1 h_2 h_3} \sum_i \frac{\partial}{\partial x^i} (h_1 h_2 h_3 M^i), \quad (7)$$

where  $e^\infty = -\alpha g_{\nu 0} T^{\nu 0}$  is called “energy-at-infinity” density, which corresponds to the total energy density of the plasma and field,  $S^i = -c\alpha g_{\nu 0} T^{i\nu}$  is the energy flux density,  $l = \alpha g_{3\nu} T^{3\nu}/c$  is the angular momentum density, and  $M^i = \alpha h_i T^{i\nu} g_{\nu 3}$  is the angular momentum flux density.

When we introduce the local frame called the “zero angular momentum observer” (ZAMO) frame, we have  $d\hat{t} = \alpha dt$ ,  $d\hat{x}^i = h_i(dx^i - \omega_i dt)$ . Because this is the local Minkowski space-time:  $ds^2 = -(cdt)^2 + \sum_{i=1}^3 (d\hat{x}^i)^2 = \eta_{\mu\nu} d\hat{x}^\mu d\hat{x}^\nu$ , the variables observed in the frame are intuitive. For example, the velocity  $\hat{v}^i$ , the Lorentz factor  $\gamma = \hat{U}^0 = [1 - \sum_{i=1}^3 (\hat{v}^i/c)^2]^{-1/2}$ , and the four-velocity  $\hat{U}^i = h_i U^i - c\alpha\beta^i U^0$  ( $i = 1, 2, 3$ ) have the relation,  $\hat{U}^i = \gamma \hat{v}^i$ . Hereafter we denote the variables observed in the ZAMO frame with the hat,  $\hat{\wedge}$ . From equation (5), we obtain

$$e^\infty = \alpha e + \sum_i \omega_i h_i \hat{P}^i = \alpha e + \omega_3 l, \quad (8)$$

$$l = h_3 \hat{P}^3. \quad (9)$$

Here  $e = \hbar\gamma^2 - p + (\hat{B}^2 + \hat{E}^2/c^2)/2$  is the total energy density and  $\hat{P}^i = \{\hbar\gamma^2 \hat{v}^i + (\hat{\mathbf{E}} \times \hat{\mathbf{B}})^i\}/c^2$  is the  $i$ -th component of the momentum density, where  $\hat{B}^i = \epsilon^{ijk} \hat{F}_{jk}/2$  and  $\hat{E}^i = c\eta^{ij} \hat{F}_{j0} = c\hat{F}_0^i = c\hat{F}_{i0}$  ( $i = 1, 2, 3$ ) are the magnetic flux density and the electric field, respectively ( $\epsilon^{\mu\nu\lambda}$  is the Levi-Civita tensor). We normalize the field strength tensor  $F_{\mu\nu}$  so that  $|\hat{\mathbf{B}}|^2/2$  and  $|\hat{\mathbf{E}}|^2/(2c^2)$  give the magnetic and electric energy densities, respectively. For example, the magnetic field measured in the SI unit divided by the square root of the magnetic permeability  $\mu_0$  is  $\hat{\mathbf{B}}$  and the electric field measured in the SI unit times the square root of permittivity of vacuum  $\epsilon_0$  is  $\hat{\mathbf{E}}$ . Equations (8) and (9) can be separated into the hydrodynamic and electromagnetic components:  $e^\infty = e_{\text{hyd}}^\infty + e_{\text{EM}}^\infty$  and  $l = l_{\text{hyd}} + l_{\text{EM}}$ , where

$$e_{\text{hyd}}^\infty = \alpha \hat{e}_{\text{hyd}} + \sum_i \omega_i h_i \frac{\hbar}{c^2} \gamma^2 \hat{v}^i = \alpha \hat{e}_{\text{hyd}} + \omega_3 l_{\text{hyd}}, \quad (10)$$

$$e_{\text{EM}}^\infty = \alpha \hat{e}_{\text{EM}} + \sum_i \omega_i h_i \frac{1}{c^2} \left( \hat{\mathbf{E}} \times \hat{\mathbf{B}} \right)_i = \alpha \hat{e}_{\text{EM}} + \omega_3 l_{\text{EM}}, \quad (11)$$

$$l_{\text{hyd}} = h_3 \frac{\hbar}{c^2} \gamma^2 \hat{v}^3, \quad (12)$$

$$l_{\text{EM}} = h_3 \frac{1}{c^2} \left( \hat{\mathbf{E}} \times \hat{\mathbf{B}} \right)_3. \quad (13)$$

Here,  $\hat{e}_{\text{hyd}} = \hbar \gamma^2 - p$  and  $\hat{e}_{\text{EM}} = (\hat{B}^2 + \hat{E}^2/c^2)/2$  are the hydrodynamic and electromagnetic energy densities observed by the ZAMO frame, respectively.

## 2.1. Penrose process

When we consider a particle with rest mass  $m$  at  $\mathbf{r}_P(t)$  in the absence of electromagnetic field, *i.e.*  $\rho = \frac{m}{\gamma} \delta^3(\mathbf{r} - \mathbf{r}_P(t))$ ,  $p = 0$ , and  $\mathbf{B} = \mathbf{E} = \mathbf{0}$ , then the energy-at-infinity and the angular momentum of the particle are given from equations (8) and (9) as,

$$E^\infty = \int_{\Omega} e^\infty dV = \alpha \gamma m c^2 + \omega_3 L, \quad (14)$$

$$L = \int_{\Omega} l dV = h_3 m \gamma \hat{v}^3, \quad (15)$$

where  $\Omega$  is the whole volume of the space. Both the energy-at-infinity and angular momentum of the particle conserve when it travels alone.

Equations (14) and (15) yield the energy-at-infinity of the particle as

$$E^\infty = \alpha \gamma m c^2 \left( 1 + \beta_3 \frac{\hat{v}^3}{c} \right). \quad (16)$$

If  $\beta_3 \hat{v}^3/c < -1$ , the energy-at-infinity of the particle becomes negative. This condition can be satisfied only in the ergosphere ( $\beta_3 > 1$ ). Using the relation  $\alpha^2 \{1 - \sum_i (\beta_i)^2\} = -g_{00}$ , we have the well-known definition of the ergosphere:  $g_{00} \geq 0$ .

When we consider the particle fission in the ergosphere,  $A \longrightarrow B + C$ , the conservation laws of the energy-at-infinity and the angular momentum are

$$E_A^\infty = E_B^\infty + E_C^\infty, \quad (17)$$

$$L_A = L_B + L_C, \quad (18)$$

$$E_I^\infty = \alpha \gamma m_I c^2 + \omega_3 L_I \quad (I = A, B, C). \quad (19)$$

If the fission is so strong that satisfies  $L_B = h_3 m_B \gamma_B \hat{v}_B^3 < -ch_3 m_B / \sqrt{(\beta_3)^2 - 1}$ , then we get the negative energy-at-infinity  $E_B^\infty < 0$  and  $E_A^\infty < E_C^\infty$ . The particle C escapes to infinity and the particle B is swallowed by the black hole to reduce the black hole mass. Eventually, the rotational energy of the black hole is extracted.

## 2.2. Blandford-Znajek mechanism

In the Blandford-Znajek mechanism, the energy propagates outwardly from the black hole horizon when the angular velocity of the black hole horizon  $\Omega_H$  is larger than that of the magnetic field lines  $\Omega_F$ :  $\Omega_H > \Omega_F$ . Although this statement looks inconsistent to the definition of the horizon, where any energy, matter, and information never transform outwardly (Punsly et al. 1990), it can be understood as the transportation of the negative electromagnetic energy-at-infinity of magnetic fields into the black hole (Koide 2003). Here we show the electromagnetic energy-at-infinity becomes negative when  $\Omega_H > \Omega_F$ .

From equations (11) and (13) and the definition of  $\hat{e}_{EM}$  and  $\hat{P}^i$ , the electromagnetic energy-at-infinity  $e_{EM}^\infty$  is written as

$$e_{EM}^\infty = \frac{\alpha}{2} \left( \hat{B}^2 + \frac{\hat{E}^2}{c^2} \right) + \alpha \boldsymbol{\beta} \cdot (\hat{\mathbf{E}} \times \hat{\mathbf{B}}). \quad (20)$$

When we assume steady-state of the electromagnetic field in the force-free condition, the electric field observed by the ZAMO frame is given by

$$\hat{\mathbf{E}} = -\frac{h_3}{\alpha} (\Omega_F - \omega_3) \mathbf{e}_\phi \times \hat{\mathbf{B}}^P, \quad (21)$$

where  $\mathbf{e}_\phi$  is the unit vector parallel to the azimuthal coordinate,  $\hat{\mathbf{B}}^P = \hat{\mathbf{B}} - \hat{B}_\phi \mathbf{e}_\phi$  is the poloidal magnetic field, and  $\Omega_F$  is a constant along the magnetic flux tube. If we use the velocity of the magnetic flux tubes observed in the ZAMO frame,  $\hat{\mathbf{v}}_F = (h_3/\alpha)(\Omega_F - \omega_3)\mathbf{e}_\phi$ , equation (21) can be written as a intuitive equation,  $\hat{\mathbf{E}} = -\hat{\mathbf{v}}_F \times \hat{\mathbf{B}}^P$ . Inserting equations (21) into (20), we have the electromagnetic energy-at-infinity,

$$e_{EM}^\infty = \alpha \frac{(\hat{B}^P)^2}{2} \left( \frac{h_\phi}{c\alpha} \right)^2 \left[ \Omega_F^2 - (\omega_3)^2 \left\{ 1 - \frac{1}{(\beta_3)^2} - \left( \frac{\hat{B}_\phi}{\beta_3 \hat{B}^P} \right)^2 \right\} \right]. \quad (22)$$

We consider the electromagnetic energy-at-infinity very near the horizon. We adopt the boundary condition of the electromagnetic field at the horizon,

$$\frac{\hat{B}_\phi}{\hat{B}^P} = \frac{\hat{v}_F^\phi(r \rightarrow r_H)}{c}, \quad (23)$$

where  $r_H$  is the radius of the horizon. The condition (23) is intuitive when we assume the Alfvén velocity is the light speed  $c$  in the force-free magnetic field. It is also identical to the condition used by Blandford & Znajek (1977), while the original expression is rather complex. After some manipulations with equations (22) and (23), we obtain at the horizon

$$\alpha e_{EM}^\infty = \left( \frac{h_3}{c} \right)^2 (\hat{B}^P)^2 \Omega_F (\Omega_F - \omega_3). \quad (24)$$

Here we have used  $\alpha \rightarrow 0$  very near the horizon. The electromagnetic energy-at-infinity is negative only when  $0 < \Omega_F < \omega_3$ , which is identical to the switch-on condition of the Blandford-Znajek mechanism. The coincidence indicates that the Blandford-Znajek mechanism utilizes the negative electromagnetic energy-at-infinity. This conclusion is generally applicable to the force-free field of any spinning black hole as far as equation (23) is valid, while the original analytical model of Blandford & Znajek (1977) is restricted to a slowly spinning black hole. In the force-free condition, the Alfvén velocity becomes the speed of light, and then the Alfvén surface is located at the horizon. Then the region of the negative energy-at-infinity can be connected with the region outside of the ergosphere through the magnetic field causally.

### 2.3. MHD Penrose process

The MHD Penrose process is the mechanism of energy extraction from a black hole through the negative energy-at-infinity of plasma induced by the magnetic tension (Hirotani et al. 1992). It has been confirmed by Koide et al. (2002) and Koide (2003) using GRMHD numerical simulations. It is defined as the energy extraction mechanism with the negative energy-at-infinity of plasma, which is induced by the magnetic tension, while in the Blandford-Znajek mechanism, the negative electromagnetic energy-at-infinity plays a important role. For a rapidly rotating black hole ( $a = 0.99995$ ), the magnetic flux tubes of the strong magnetic field which cross the ergosphere are twisted due to the frame-dragging effect. The angular momentum of plasma in the ergosphere is opposite to that of the rotating black hole and its magnitude is large to make  $e_{\text{hyd}}^\infty$  negative in equation (10). The twist of the magnetic flux tubes propagates outwardly. The Poynting flux indicates that the electromagnetic energy is radiated from the ergosphere (see figure 4 of Koide et al. 2003). At the foot point of the energy radiation from the ergosphere, the hydrodynamic energy-at-infinity  $e_{\text{hyd}}^\infty = \alpha\gamma\hat{e}_{\text{hyd}} + \omega_3 l_{\text{hyd}}$  decreases rapidly and becomes negative quickly. The negative energy-at-infinity is mainly composed of that of the plasma. To realize the negative energy-at-infinity of the plasma, redistribution of the angular momentum of the plasma,  $l_{\text{hyd}} = h_3\hbar\gamma^2\hat{v}_\phi$ , is demanded. The angular momentum of the plasma is mainly redistributed by the magnetic tension.

## 3. Magnetic reconnection in ergosphere

We investigate energy extraction through negative energy-at-infinity induced by magnetic reconnection in an ergosphere around a Kerr black hole. For simplicity, we consider the magnetic reconnection in the bulk plasma rotating around the black hole circularly at the

equatorial plane (Fig. 1). To sustain the circular orbit, the plasma rotates with the Kepler velocity  $v_K$  or it is supported by external force, like magnetic force. Here the Kepler velocity is given by

$$\hat{v}_K = \frac{cA \left[ \pm \sqrt{r_g/r} - ar_g^2/r^2 \right]}{\sqrt{\Delta(r^3 - r_g^3 a^2)}} - c\beta^3. \quad (25)$$

The plus (minus) sign corresponds to the co-rotating (counter-rotating) circular orbit case. Throughout this paper, we use the co-rotating Kepler velocity. We assume the initial anti-parallel magnetic field directs to the azimuthal direction in the bulk plasma, and the pair plasma outflows moving toward the opposite directions each other caused by the magnetic reconnection are ejected in the azimuthal direction. It is also assumed that the plasma acceleration through the magnetic reconnection is localized in the very small region compared to the size of the black hole ergosphere, and the magnetic field outside of the plasma acceleration region is so weak that the plasma flow accelerated by the magnetic reconnection is not influenced by the large-scale magnetic field around the black hole. If one of the pair plasma flows in the opposite direction of the black hole rotation has negative energy-at-infinity and the other in the same direction of the black hole rotation has the energy-at-infinity higher than the rest mass energy (including the thermal energy) (Fig. 1), the black hole rotational energy will be extracted just like the Penrose process.

We have to investigate two conditions, *i.e.* the condition for the formation of the negative energy-at-infinity and the condition for escaping to infinity. Before we move on to the conditions, we here mention about the elementary process of the relativistic magnetic reconnection in the locally uniform, small-scale plasma rotating circularly around the black hole. To investigate the magnetic reconnection in the small scale, we introduce the local rest frame  $(ct', x^{1'}, x^{2'}, x^{3'})$  of the bulk plasma which rotates with the azimuthal velocity  $\hat{v}^3 = c\beta_0$  at the circular orbit on the equatorial plane  $r = r_0 < r_S$ ,  $\theta = \pi/2$ . We set the frame  $(ct', x^{1'}, x^{2'}, x^{3'})$  so that the direction of  $x^{1'}$  coordinate is parallel to the radial direction  $x^1 = r$  and the direction of  $x^{3'}$  is parallel to the azimuthal direction  $x^3 = \phi$  (Fig. 1). Hereafter we note the variables observed by the rest frame of the bulk plasma with the prime, “ $'$ ”. First, we consider the magnetic reconnection in the rest frame locally and neglect the tidal force and Coriolis’ force for simplicity. We regard that the rest frame rotating with the Kepler velocity is in a gravity-free state and thus we can consider the magnetic reconnection in the framework of special relativistic MHD. The initial condition is set to that of the Harris model where the anti-parallel magnetic field and the plasma are in equilibrium:

$$B^{3'} = B_0 \tanh(x^{1'}/\delta), \quad B^{1'} = B^{2'} = 0, \quad (26)$$

$$p = \frac{B_0^2}{2 \cosh^2(x^{1'}/\delta)} + p_0, \quad (27)$$

$$\rho = \rho_0, \quad (28)$$

$$v^{1'} = 0, \quad v^{2'} = 0, \quad v^{3'} = 0, \quad (29)$$

where  $B_0$  is the typical magnetic field strength and  $2\delta$  is the thickness of the current layer (see Fig. 2). The electric resistivity is assumed to be zero except for the narrow reconnection region. We assume the system is symmetric with respect to the  $x^{2'}$ -direction, and resistivity is given by

$$\eta = \eta_0 f(x^{1'}, x^{3'}), \quad (30)$$

where  $\eta_0$  is a positive constant and  $f(x^{1'}, x^{3'})$  is the profile of the resistivity, which is finite around  $x^{1'} = 0, x^{3'} = 0$  but zero outside the reconnection region. The magnetic flux tubes reconnected at the resistive region accelerate the plasma through the magnetic tension, and the magnetic energy in the flux tubes is converted to the kinetic energy of the plasma. As shown in Fig. 3, the magnetic flux tube and the plasma run away and fresh tubes and the plasma are supplied to the reconnection region from outside of the current layer successively.

The outflow velocity of the accelerated plasma through the magnetic reconnection,  $v'_{\text{out}}$ , is estimated by the velocity  $v'_{\text{max}}$ , where whole magnetic energy is converted to the kinetic energy. If the magnetic energy is completely converted to the kinetic energy of the plasma particles which are initially at rest, the magnetic energy per plasma particle is  $B_0^2/(2n_0)$ , where  $n_0$  is the plasma particle number density. Using the approximation that the plasma element is treated as incompressible gas covered by very thin, light, adiabatic skin with the thermal energy  $U$  and the enthalpy  $H$ , using equation (A5) in Appendix A, we can write the energy conservation equation with respect to the particle as,

$$\gamma'_{\text{max}} H - \frac{(\Gamma - 1)U}{\gamma'_{\text{max}}} = \frac{B_0^2}{2n_0} + H - (\Gamma - 1)U, \quad (31)$$

where  $\gamma'_{\text{max}}$  is the Lorentz factor of the plasma particle after complete release of the magnetic field energy observed in the bulk plasma rest frame. We get the maximum Lorentz factor  $\gamma'_{\text{max}}$  from equation (31) as,

$$\gamma'_{\text{max}} = \frac{1}{4} \left[ u_A^2 + 2(1 - \varpi) + \sqrt{4D} \right], \quad (32)$$

where  $u_A^2 = B_0^2/h_0$ ,  $\varpi = p_0/h_0$ ,  $4D = [u_A^2 + 4(1 - \varpi)]^2 + 16\varpi$ ,  $p_0 = (\Gamma - 1)U n_0$  is the pressure, and  $h_0 = n_0 H$  is the enthalpy density of the plasma. Obviously, the terminal velocity of the plasma outflow through the magnetic reconnection is smaller than the maximum velocity  $v'_{\text{max}}$ , since all the magnetic energy is not always converted to the kinetic energy because of the Joule heating in the reconnection region and finite length acceleration. Figure 4 shows the four-velocity  $u'_{\text{out}} = \gamma'_{\text{out}} v'_{\text{out}}$  of the plasma outflow caused by the magnetic reconnection

against the plasma beta  $\beta_P = 2p_0/B_0^2$  in the case of  $\Gamma = 4/3$  and  $B_0^2 = \rho_0 c^2$ . The solid line denotes the predicted values from equation (32),

$$u'_{\max} = \gamma'_{\max} \frac{v'_{\max}}{c} = \frac{1}{2} \left[ \left( 1 + \frac{u_A^2}{2} - \varpi \right)^2 + 2(\varpi - 1) + \left( 1 + \frac{u_A^2}{2} - \varpi \right) \sqrt{D} \right], \quad (33)$$

where

$$u_A = \left[ \frac{\rho_0 c^2}{B_0^2} + \frac{\Gamma}{2(\Gamma - 1)} \beta_P \right]^{-1/2}, \quad \varpi = \frac{\beta_P}{2} \left[ \frac{\rho_0 c^2}{B_0^2} + \frac{\Gamma}{2(\Gamma - 1)} \beta_P \right]^{-1}. \quad (34)$$

The full squares are from the numerical calculations (Watanabe et al. 2006). It is found that our values of  $u'_{\max}$  are in good agreement with the result of numerical simulations. In the high plasma beta region, the values of  $u'_{\max}$  from equation (33) are slightly smaller than those of the numerical result. This discrepancy, *i.e.*  $u'_{\max} < u'_{\text{out}}$ , should come from the release of the thermal energy to the plasma kinetic energy. Therefore, we take the maximum velocity  $v'_{\max}$  as the plasma outflow velocity induced by the magnetic reconnection hereafter.

Now we return to consider the conditions of the negative energy-at-infinity formation and escaping to infinity of the pair outflows caused by the magnetic reconnection. From equations (A6), the hydrodynamic energy-at-infinity per enthalpy of the plasma ejected through the magnetic reconnection into the  $\pm x^{3'}$  (azimuthal) direction with the Lorentz factor  $\gamma'_{\max}$  is given by

$$\begin{aligned} \epsilon_{\pm}^{\infty}(u_A, \hat{\beta}_0, \varpi, \alpha, \beta_3) &\equiv \frac{e_{\text{hyd}, \pm}^{\infty}}{h_0} = \frac{E_{\pm}^{\infty}}{H} \\ &= \alpha \hat{\gamma}_0 \left[ (1 + \beta_3 \hat{\beta}_0) \gamma'_{\max} \pm (\hat{\beta}_0 + \beta_3) \sqrt{(\gamma'_{\max})^2 - 1} - \frac{\gamma'_{\max} \mp \hat{\beta}_0 \sqrt{(\gamma'_{\max})^2 - 1}}{(\gamma'_{\max})^2 + \hat{\gamma}_0^2 \hat{\beta}_0^2} \varpi \right], \end{aligned} \quad (35)$$

where  $\gamma'_{\max}$  is given by equation (32). The plus and minus signs of the subscript in  $\epsilon_{\pm}^{\infty}$  express the cases with the plasma velocity  $v^{3'} = v'_{\max}$  and  $v^{3'} = -v'_{\max}$ , respectively. If  $\epsilon_{-}^{\infty} < 0$ , the energy-at-infinity of the plasma becomes negative. Here we neglect the contribution from the electromagnetic field, because when the plasma velocity becomes  $v'_{\max}$ , most of the magnetic energy is converted to the plasma kinetic energy so that the electromagnetic energy-at-infinity becomes negligible. Then the total energy-at-infinity of the plasma and the electromagnetic field becomes negative. Here  $\epsilon_{-}^{\infty}$  decreases monotonically when  $u_A$  increases, and it is positive when  $u_A = 0$  and negative when  $u_A$  is large enough in the ergosphere. Then, we can define the one-valued function  $U_A^{-}(\hat{\beta}_0, \varpi, \alpha, \beta_3)$  which satisfies  $\epsilon_{-}^{\infty}(U_A^{-}, \hat{\beta}_0, \varpi, \alpha, \beta_3) = 0$ . The condition for the formation of the negative energy-at-infinity through the magnetic reconnection is given by  $u_A > U_A^{-}$ .

The condition of escaping to infinity of the plasma particle accelerated by the magnetic

reconnection is given by

$$\Delta\epsilon_+^\infty \equiv \epsilon_+^\infty - \left(1 - \frac{\Gamma}{\Gamma-1}\varpi\right) > 0. \quad (36)$$

Here we assume the magnetic field is so weak far from the reconnection region that we can neglect the interaction between the large-scale magnetic field and the outflow from the reconnection region. The monotonic function  $\Delta\epsilon_+^\infty$  with respect to  $u_A$  is negative when  $u_A = 0$  and positive when  $u_A$  is large enough. Then we can also define the one-valued critical function  $U_A^+(\hat{\beta}_0, \varpi, \alpha, \beta_3)$  such that  $\Delta\epsilon_+^\infty(U_A^+, \hat{\beta}_0, \varpi, \alpha, \beta_3) = 0$ . The condition of the escape to infinity of the plasma flow is written by  $u_A > U_A^+$ .

We apply the conditions  $u_A > U_A^+$  and  $u_A > U_A^-$  to the plasma with the rotation bulk velocity  $\hat{v}^3 = c\hat{\beta}_0 = \hat{v}_K$ . Figure 5 shows  $U_A^-$  and  $U_A^+$  against  $r/r_S$  ( $r_S = 2r_g$ ) for the fixed black hole rotation parameter  $a = 0.995$  and the pressure parameter  $\varpi = 0, 0.2, 0.5$ . The lines of the critical values of Alfvén four-velocities  $U_A^-$  and  $U_A^+$  are drawn as the contours of  $\epsilon_-^\infty = 0$  and  $\Delta\epsilon_+^\infty = 0$ , respectively. The vertical thin dashed line indicates the inner most stable orbit radius of a single particle. The vertical thin dot-dashed line shows the point of  $\hat{v}_K = c$ . Then, the Kepler motion is unstable between the dot-dashed line and the dashed line. In the left region of the dot-dashed line, there is no circular orbit anymore. The vertical thin solid line indicates the horizon of the black hole. The upper regions of the thick lines  $\epsilon_-^\infty = 0$  show the condition of the formation of the negative energy-at-infinity through the magnetic reconnection,  $u_A = B_0/\sqrt{h_0} > U_A^-$ . The different styles of lines indicate the different pressure cases (solid line:  $\varpi = 0$ , dashed line:  $\varpi = 0.2$ , dot-dashed line:  $\varpi = 0.5$ ). The regions above the thick lines  $\Delta\epsilon_+^\infty = 0$  show the condition on escaping of the plasma accelerated by the magnetic reconnection,  $u_A = B_0/\sqrt{h_0} > U_A^+$ . The difference of the line styles denotes the same as that of the upper lines. It is shown that the condition of the energy extraction from the black hole through the magnetic reconnection is determined from the condition of the formation of the negative energy-at-infinity,  $u_A > U_A^-$  in the case of the rotating plasma with the Kepler velocity. In the zero pressure case ( $\varpi = 0$ ), the easiest condition is found at  $r = 0.61r_S$ ,  $u_A \geq U_A^- = 0.86$ . This means the relativistic reconnection is required for the energy extraction from the black hole. In the finite pressure case ( $\varpi = 0.2$ ), the condition is relatively relaxed around the outer region of the ergosphere, while the condition around the inner ergosphere is severer. However, the difference is small between these cases. This case also requires the relativistic magnetic reconnection to extract the black hole energy. The condition of the case  $\varpi = 0.5$  is almost similar to the tendency of the previous two cases ( $\varpi = 0, 0.2$ ).

Figure 6 shows the critical Alfvén four-velocity  $U_A^\pm$  of the energy extraction from the black hole with the rotation parameter  $a = 0.9$ . In this case, the condition of the energy extraction from the black hole becomes severer compared to the cases with the larger rotation

parameters  $a = 0.995$ .

It is noted that in the case of the black hole rotation parameter  $a < 1/\sqrt{2}$ , there is no circular orbit inside of the ergosphere. In such case, we can not consider the energy extraction through the magnetic reconnection in the circularly rotating plasma around the black hole except for the case with the support by magnetic field.

When we consider the slower rotating plasma case  $c\hat{\beta}_0 < \hat{v}_K$ , which may be an artificial assumption, the conditions of the negative energy-at-infinity and the escape plasma become comparable and relaxed as a whole. Figure 7 shows the critical Alfvén four-velocity of the case  $c\hat{\beta}_0 = 0.6\hat{v}_K$ ,  $a = 0.995$ . At  $r = 0.6r_S$ ,  $U_A^+ \simeq U_A^- \simeq 0.5$  for both the  $\varpi = 0$  and  $\varpi = 0.2$  cases. This means the sub-relativistic magnetic reconnection can extract the black hole energy.

#### 4. Discussion

In the previous section, we showed the possibility of the energy extraction from the black hole through the magnetic reconnection in the ergosphere. In this mechanism, the magnetic tension plays a significant role to cause the plasma flow with the negative energy-at-infinity, like the MHD Penrose process. If we consider quick magnetic reconnection, the mechanism by the magnetic reconnection is more effective than the MHD Penrose process, because the fast plasma flow can be induced, so that all magnetic energy can be converted to the kinetic energy of the plasma flow through the magnetic reconnection.

As the magnetic reconnection mechanism, we utilized a rather simple model with an artificial resistivity at the reconnection region and the local approximation around the rotating black hole. We further assumed that the outflow caused by the magnetic reconnection is parallel to the azimuthal direction. In general, the outflow is oblique to the azimuthal direction. In the oblique case with the angle  $\chi$  between the outflow and azimuthal directions, the condition of the energy extraction through the magnetic reconnection is given by  $u_A > U_A^-(u_A, \varpi, \alpha, \beta_3 \cos \chi)$  and  $u_A > U_A^+(u_A, \varpi, \alpha, \beta_3 \cos \chi)$ , where  $\beta_3$  of equation (35) is replaced by  $\beta_3 \cos \chi$ . This is a severer condition compared to the parallel case.

Let us briefly estimate the critical magnetic field required for the energy extraction from the black hole through the magnetic reconnection with respect to the AGNs,  $\mu$ QSOs, and GRBs. Here the critical magnetic field in the SI unit,  $B_{\text{crit}}$ , is given by  $B_{\text{crit}} = \sqrt{\mu_0 h_0} \sim \sqrt{\mu_0 \rho_0} c$ , where  $h_0$  and  $\rho_0$  are the typical enthalpy and mass density around the objects, respectively. Here, we assume the pressure is not larger than  $\rho_0 c^2$  and neglect it to get rough estimation. To estimate the typical mass density  $\rho_0$  of the plasma around the central black

hole of these objects, we use

$$\rho_0 \simeq 5 \times 10^4 \left( \frac{\dot{M}}{M_{\odot} \text{yr}^{-1}} \right) \left( \frac{M}{M_{\odot}} \right)^{-2} \left( \frac{2r}{r_{\text{S}}} \right)^{-3/2} \text{ g cm}^{-3}, \quad (37)$$

where  $\dot{M}$  is the accretion rate and  $M$  is the black hole mass (Rees 1984). For the AGN in the large elliptical galaxy M87, when we assume  $\dot{M} = 10^{-2} M_{\odot} \text{yr}^{-1}$ ,  $r = r_{\text{S}}$ , and  $M = 3 \times 10^9 M_{\odot}$  (Reynolds et al. 1996; Ho 1999), equation (37) yields  $\rho_0 \simeq 2 \times 10^{-17} \text{ g cm}^{-3}$ . Then, we get  $B_{\text{crit}} \simeq 500 \text{ G}$ . This magnetic field is probable around a black hole of an AGN, thus the magnetic extraction of black hole energy is possible.

In a collapsar model with  $\dot{M} = 0.1 M_{\odot} \text{s}^{-1}$  and  $M = 3M_{\odot}$  (MacFadyen et al. 1999), when we apply equation (37), we get  $\rho_0 \simeq 6 \times 10^9 \text{ g cm}^{-3}$  at  $r = r_{\text{S}}$  as mass density around a black hole in a GRB progenitor. The critical magnetic field is then  $B_{\text{crit}} \simeq 8 \times 10^{15} \text{ G}$ . The magnetic field of the progenitors is estimated to be  $10^{15} \text{ G}$  to  $10^{17} \text{ G}$  (van Putten 1999) and then extraction of the black hole energy through the magnetic reconnection is marginally probable in a core of a collapsar.

With respect to  $\mu$ QSO, GRS1915+105 has a mass accretion rate of  $\dot{M} = 7 \times 10^{-7} M_{\odot} \text{yr}^{-1}$  (Mirabel et al. 1994; Fender et al. 2004) with a mass of  $M = 14M_{\odot}$  (Greiner et al. 2001). Then equation (37) yields  $\rho_0 \simeq 6 \times 10^{-5} \text{ g cm}^{-3}$ . The critical magnetic field is estimated as  $B_{\text{crit}} \simeq 8 \times 10^8 \text{ G}$ . This magnetic field is too strong as the field around a black hole in  $\mu$ QSOs. Thus, the energy extraction from a black hole in  $\mu$ QSO through the magnetic reconnection may not be prospective.

We discuss the possibility of formation of anti-parallel magnetic field with a current sheet where strong magnetic reconnection in the ergosphere is caused. First, let us consider uniform magnetic field around a rotating black hole as the initial condition. In this magnetic configuration, one may think that the magnetic reconnection scarcely happens. However, this is caused naturally by the gravitation and the frame-dragging effect of the rapidly rotating black hole. Under this situation, GRMHD simulations were carried out with zero electric resistivity (Koide et al. 2002; Koide 2003; Komissarov 2004). The numerical simulations showed that the magnetic flux tubes across the ergosphere are twisted by the frame-dragging effect of the rotating black hole, and the plasma falling into the black hole makes the magnetic field radial around the ergosphere. Here it is noted that the magnetic field line twisted by the frame-dragging effect makes the angular momentum of the plasma around the equatorial plane and the ergosphere negative ( $l < 0$ ), and the plasma with the negative angular momentum falls into the black hole more rapidly. The attractive force toward the black hole comes from the shear of the frame-dragging effect directly (e.g. see the term with  $\sigma_{ji}$  in equation (56) of Koide (2003)). Beside the equatorial plane in the

ergosphere, the magnetic field becomes anti-parallel. The magnetic flux tubes are twisted strongly enough, and then the strong anti-parallel open magnetic field is formed almost along the azimuthal direction (Fig. 8a). In this way, the magnetic reconnection happens and the energy of the rotating black hole is extracted through the magnetic reconnection, even in the case of the initially uniform magnetic field. When the magnetic reconnection happens around the anti-parallel magnetic field, the outward flow from the reconnection region will be ejected toward infinity along the open magnetic field lines. This outflow will be bent and pinched by the magnetic field and may become a jet.

Next, as the initial condition, we assume closed magnetic flux tubes which are believed to be formed in the accretion disks around the black holes (van Putten 1999; Koide et al. 2006; McKinney 2006) (Fig. 8b). When a single closed magnetic flux tube is tied to an edge of a rotating quasi-stationary disk and a bulk part of the disk, the plasma at the edge loses the angular momentum and falls into the black hole, while the bulk plasma tied to the magnetic flux tube increases the angular momentum and shifts outwardly. The plasma at the disk edge falls spirally due to the frame-dragging effect, and the magnetic flux tubes dragged by the plasma are elongated spirally. If the twist of the magnetic flux tube is strong enough, anti-parallel closed magnetic field is formed almost along the azimuthal direction. In this magnetic configuration, energy may be extracted from the black hole through the magnetic reconnection in the ergosphere. The outflow from the reconnection region will elongate the closed magnetic field lines, while this decelerates the outflow. If the magnetic reconnection is caused in the elongated magnetic field lines, the plasmoid is formed and is ejected to infinity.

As shown in the above two cases of the open and closed magnetic field, the large-scale dynamics of the outflow through the magnetic reconnection depends on the large-scale magnetic field configuration. Here we note that closed magnetic flux tubes across an accretion disk and an ergosphere around a rapidly rotating black hole is unstable and expands vertically to form open magnetic field (Koide et al. 2006; McKinney 2006). Then, around a rapidly rotating black hole, open magnetic field may be probable compared to closed field. Anyway, these phenomena should be investigated by numerical simulations of the full GRMHD with non-zero electric resistivity (resistive GRMHD).

To be more exact, in both cases of the open and closed topologies of the magnetic field, the outflow caused by the magnetic reconnection is influenced by the large-scale magnetic field, and it is not determined only from the local approximation which we used here. In both cases, it is noted that the plasma with the negative energy-at-infinity is farther from the horizon than the plasma accelerated by the magnetic reconnection. If we assume the azimuthal symmetry of the initial condition, interchange instability should be caused so that the plasma with negative energy-at-infinity falls into the black hole and the plasma

with additional energy-at-infinity runs away to infinity. With respect to the interchange instability, we consider essentially hydrodynamic mode where initially super-Keplerian inner part of the disk supports sub-Keplerian outer part against the black hole gravity. The closed magnetic flux tube is formed across the plasma with additional energy-at-infinity. The plasma at the inner edge of the magnetic loop falls into the black hole because of the deceleration by the magnetic tension, while the plasma at the outer edge of the magnetic loop is accelerated and escapes to infinity. Then the magnetic flux tube is elongated between the escaping and falling plasmas. In such magnetic flux tube, the anti-parallel magnetic field with strong current sheet may be formed and the magnetic reconnection may be caused once again. On the other hand, the plasma with the negative energy-at-infinity through the magnetic reconnection falls into the black hole and the magnetic flux tube tied to the plasma is also elongated by the frame-dragging effect. The anti-parallel magnetic field in the magnetic flux tube will also form and the magnetic reconnection is caused repeatedly. Above discussion shows that the magnetic reconnection can be caused intermittently in the ergosphere. To investigate these phenomena, the numerical simulations of resistive GRMHD are also demanded.

The resistive GRMHD should solve the problems with respect to the energy extraction through the magnetic reconnection. For example, using resistive GRMHD simulations, we can take into account of the situation that the initial plasma falls into the black hole with sub-Keplerian velocity of the plasma rotation, which is neglected in this paper. Unfortunately, no simulation with resistive GRMHD has been performed until now, while recently, ideal GRMHD numerical simulations, where the electric resistivity is zero, have come spread (Koide et al. 1998, 1999, 2000, 2002, 2006; Koide 2003, 2004; McKinney 2006; Punsly 2006; Komissarov et al. 2007, and references therein). They confirmed important, interesting magnetic phenomena around the rotating black holes, such as magnetically-induced energy extraction from the rotating black hole (Koide et al. 2002; Koide 2003; Komissarov 2004) and formation of magnetically-driven relativistic jets (McKinney 2006). To confirm that the plasma with additional energy by the magnetic reconnection escapes to infinity and the plasma with the negative energy-at-infinity falls into the black hole, we may also use the ideal GRMHD simulations. In such calculations, we can trace the plasma trajectories after the magnetic reconnection stops. The magnetic configuration of the post stage of the magnetic reconnection is used as an initial condition.

On the other hand, in spite of the restriction of the ideal GRMHD, many magnetic islands are seen in the last stages of long-term calculations (McKinney et al. 2004; Koide et al. 2006; McKinney 2006). These magnetic islands, of course, are artificial appearance. However, these numerical results indicate that the magnetic configuration where the magnetic reconnection occurs is relatively easily formed around the black hole. Recent X-ray observations

of the solar corona confirmed that the magnetic reconnection takes places frequently in the active region of the corona and causes drastic phenomena, like solar flares. The observations and the recent theories of solar and stellar flares indicate that the magnetic reconnection is common in the astrophysical plasmas around the Sun, stars, and the compact objects including black holes (Masuda 1994; Shibata 1997). The energy extraction from the black hole through the magnetic reconnection is one of the phenomena of magnetic reconnection around the black hole. More drastic phenomena related with the magnetic reconnection may exist. To investigate these phenomena, resistive GRMHD numerical calculations will play a crucial role.

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### A. Relativistic adiabatic incompressible ball approach

To take account of inertia effect of pressure into plasma, we use an approximation of incompressible fluid, which is consisted of small, separated, constant volume elements. The element is covered by thin, light, adiabatic, closed skin and its volume is constant, like a ball used for soft tennis. We call this method “relativistic adiabatic incompressible ball (RAIB) approach”. Here we assumed gas pressure does not work to the plasma and influences the plasma dynamics only through an inertia effect. Let us consider the fluid in one ball with the mass  $m$ . When the ball locates at  $\mathbf{r} = \mathbf{r}(t)$ , the mass density of the gas is

$$\rho(\mathbf{r}, t) = \frac{m}{\gamma(t)} \delta^3(\mathbf{r} - \mathbf{r}(t)), \quad (\text{A1})$$

where  $\gamma(t)$  is the Lorentz factor of the ball at time  $t$  and  $\delta^3(\mathbf{r})$  is the Dirac's  $\delta$ -function in three-dimensional space. Because the gas in the ball is assumed to be incompressible and adiabatic and then its temperature is constant, the pressure should be proportional to the mass density,  $p(\mathbf{r}, t) \propto \rho(\mathbf{r}, t)$ . On the other hand, the thermal energy in the ball

$$U = \int_{\Omega} \frac{p(\mathbf{r}, t)}{\Gamma - 1} \gamma(t) dV \quad (\text{A2})$$

should be constant. Then we found

$$p(\mathbf{r}, t) = \frac{(\Gamma - 1)U}{\gamma(t)} \delta^3(\mathbf{r} - \mathbf{r}(t)). \quad (\text{A3})$$

Using equations (10), (A1), and (A3), we found the energy-at-infinity of the gas in the ball as,

$$\begin{aligned}
 E^\infty &= \int_{\Omega} \left( \hat{e}_{\text{hyd}} + \sum_i c\beta_i \frac{\hbar}{c^2} \hat{\gamma}^2 \hat{v}^i \right) dV \\
 &= \int_{\Omega} \alpha \left[ \left( \rho c^2 + \frac{\Gamma}{\Gamma-1} p \right) \hat{\gamma}^2 (1 + \beta_3 \hat{\beta}^3) - p \right] dV \\
 &= \alpha \left[ \left( \hat{\gamma} + \beta_3 \hat{U}^3 \right) H - \frac{\Gamma-1}{\hat{\gamma}} U \right], \tag{A4}
 \end{aligned}$$

where  $\hat{\beta}^3 = \hat{v}^3/c$  and  $H = mc^2 + \Gamma U$ . In the special relativistic case, equation (A4) yields the total energy of the ball as,

$$E_{\text{tot}} = \gamma H - \frac{\Gamma-1}{\gamma} U. \tag{A5}$$

Next we derive the energy-at-infinity of the incompressible ball rotating circularly around the black hole. We assume that the bulk plasma rotates circularly with the velocity  $\hat{v}^3 = c\hat{\beta}_0$ , and then the relative three-velocity between the rest frame of the bulk plasma and the ZAMO frame is  $\hat{v}^3 = c\hat{\beta}_0$ ,  $\hat{v}^1 = \hat{v}^2 = 0$ . The line elements of the ZAMO frame  $(ct, \hat{x}^1, \hat{x}^2, \hat{x}^3)$  and the bulk plasma rest frame  $(ct', x^{1'}, x^{2'}, x^{3'})$  are related by the Lorentz transformation. Using equations (A4) and the Lorentz transformation, the energy-at-infinity of the incompressible ball with four-velocity observed by the bulk plasma rest frame  $(\gamma', U'^1, U'^2, U'^3)$  is given by

$$E^\infty = \alpha H \hat{\gamma}_0 \left[ \left( 1 + \beta_3 \hat{\beta}_0 \right) \gamma' + \left( \hat{\beta}_0 + \beta_3 \right) U'^3 - \frac{\gamma' - \hat{\beta}_0 U'^3}{\gamma'^2 + \hat{\gamma}_0^2 \hat{\beta}_0^3} \varpi \right]. \tag{A6}$$

It is noted that the four-velocity of the plasma is related with the Lorentz factor by  $U'^3 = \pm\sqrt{\gamma'^2 - 1}$  in the case of  $U'^1 = U'^2 = 0$ . This formula of the energy-at-infinity of the incompressible ball (A6) is applicable to that of one particle of the plasma effectively.

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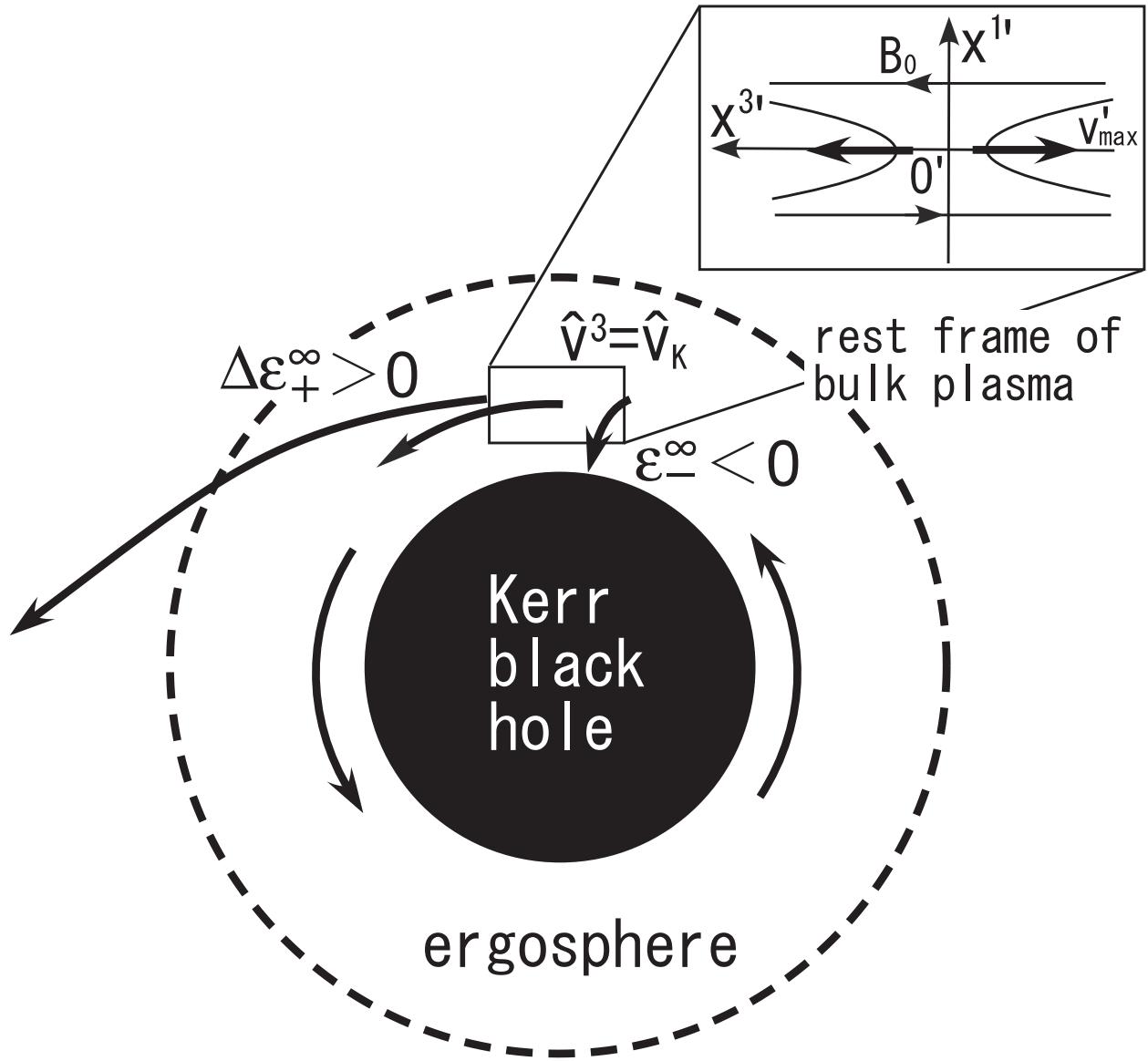


Fig. 1.— Schematic picture of the extraction of the black hole energy through the magnetic reconnection in the bulk plasma which rotates circularly on the equatorial plane of the rotating black hole. The phenomena are indicated in the Boyer-Lindquist coordinates. The coordinates  $O' - x'^1 x'^2 x'^3$  in the inserted box are the rest frame of the bulk plasma.

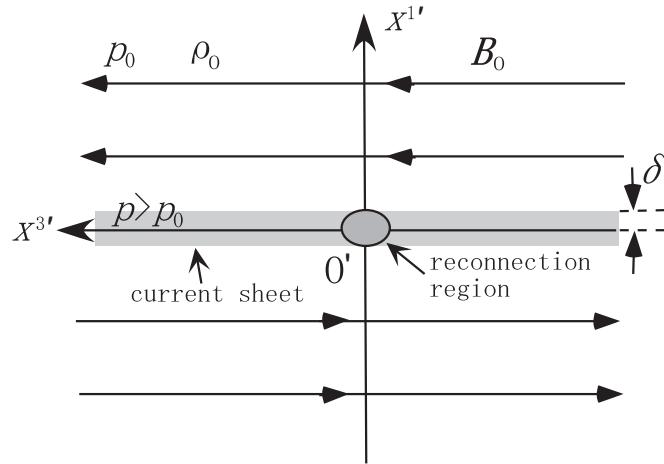


Fig. 2.— Initial magnetic configuration around the reconnection region in the local rest frame of the bulk plasma. The area in the dark gray ellipse at the origin corresponds to the magnetic reconnection region and the gray layer along the  $x^{3'}$  shows the current sheet.

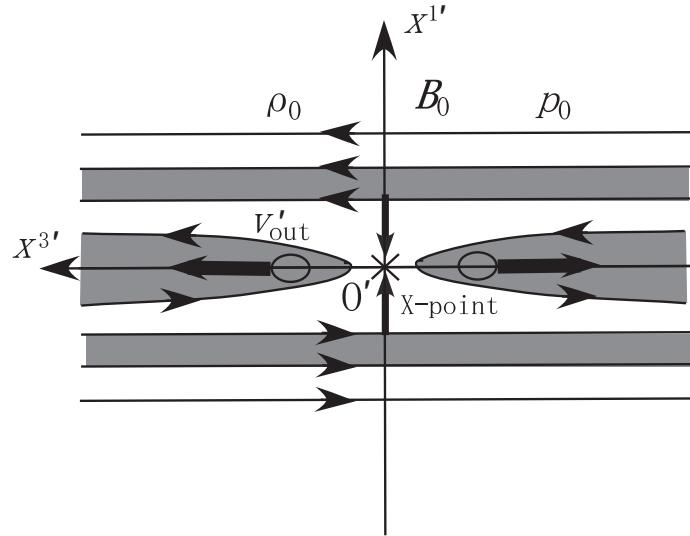


Fig. 3.— Energy convergence from the magnetic energy to the kinetic energy through the magnetic reconnection in the rest frame of the bulk plasma.

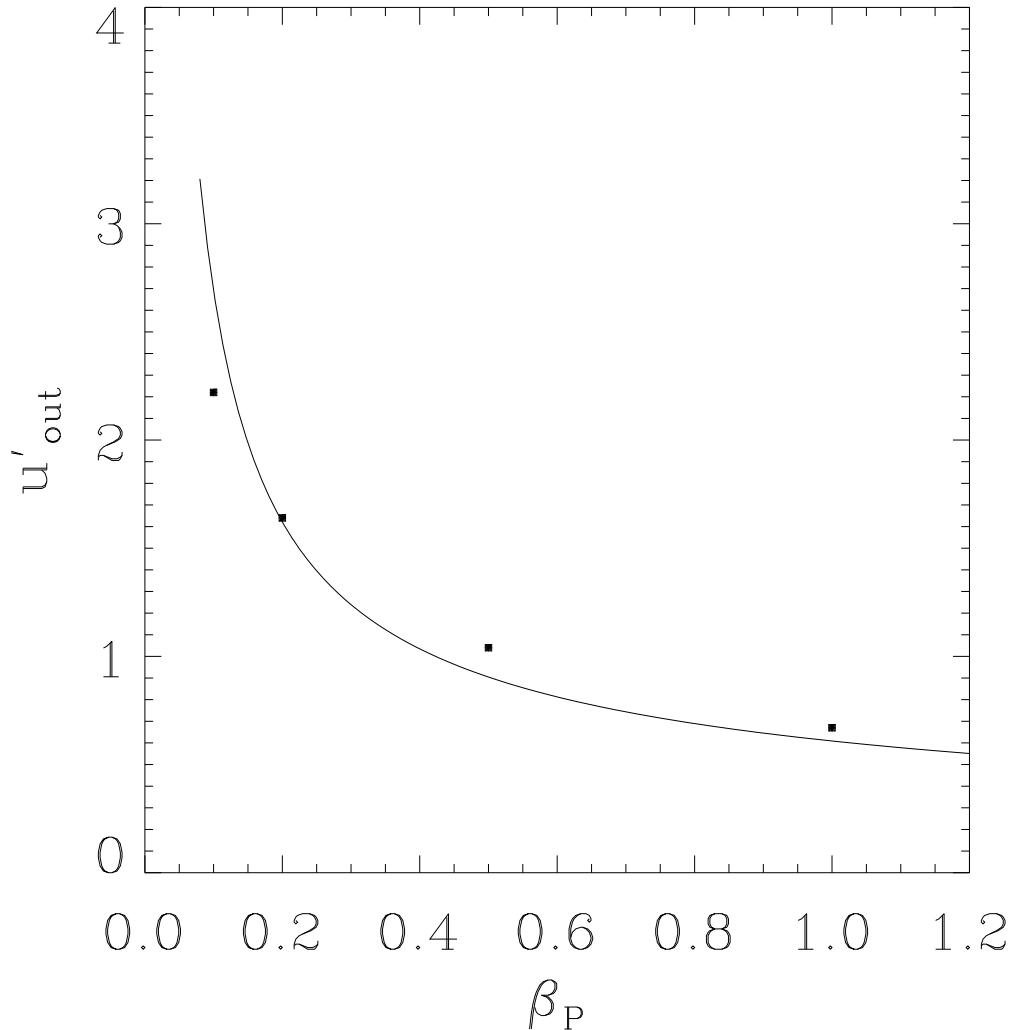


Fig. 4.— Comparison between the numerical result of the four-velocity of the plasma outflow  $u'_\text{out}$  through the magnetic reconnection (full squares; Watanabe et al. 2006) and the simple expression (33) (solid line) as a function of the plasma beta,  $\beta_P$ .

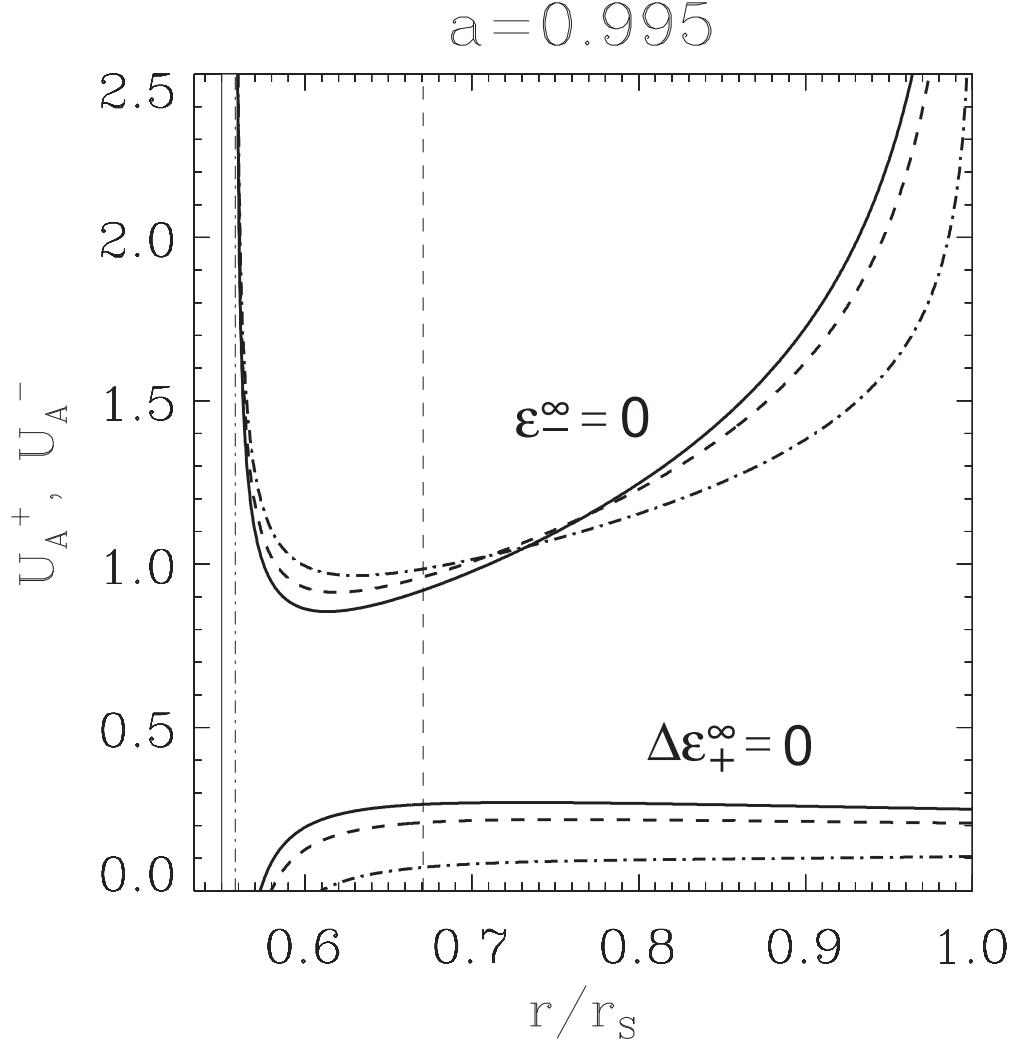


Fig. 5.— The critical Alfvén four-velocity  $U_A^\pm$  for the energy extraction from the rotating black hole with the rotation parameter  $a = 0.995$  induced by the magnetic reconnection. The base plasma rotates around the black hole with the Kepler velocity  $c\hat{\beta}_0 = \hat{v}_K$ . The thick solid, dashed, and dot-dashed lines correspond to the cases of  $\varpi = 0, 0.2, 0.5$ , respectively. The lines with the sign  $\Delta\epsilon_+^\infty = 0$  ( $\epsilon_-^\infty = 0$ ) show the critical Alfvén four-velocity of the condition on escaping to infinity of the plasma accelerated by the magnetic reconnection (the negative energy-at-infinity plasma formation). The vertical thin solid line at  $r_H = 0.550r_S$  shows the black hole horizon. The vertical thin dashed line at  $r_{ms} = 0.671r_S$  indicates the radius of the marginal stable orbit. The vertical thin dot-dashed line at  $r_L = 0.559r_S$  shows the point where the Kepler velocity is the light velocity  $c$ . Between the vertical thin solid line and the vertical thin dot-dashed line, there is no circular orbit of a particle. Generally speaking,  $U_A^-$  is infinite at  $r = r_S$ , while  $U_A^+$  is finite at this point.

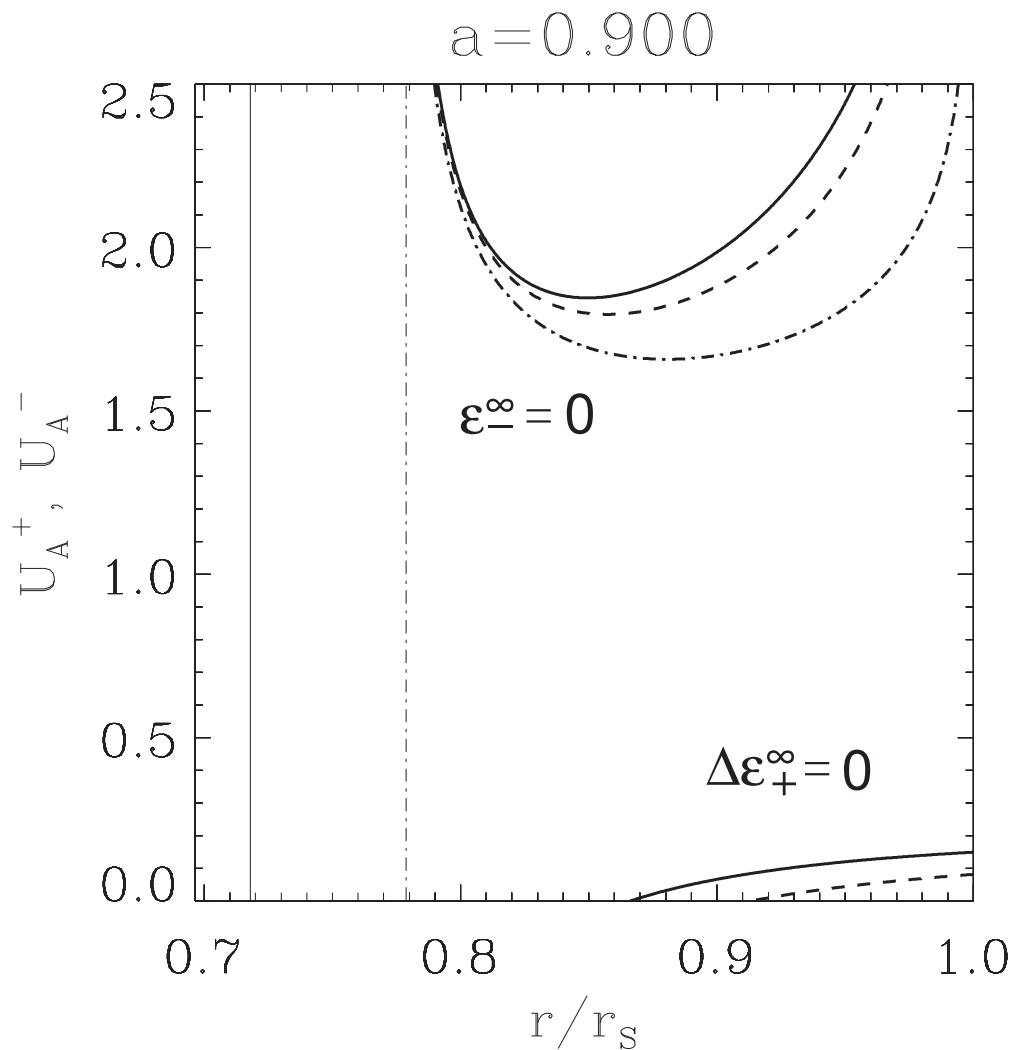


Fig. 6.— Similar to Fig. 5, but for the black hole with the rotation parameter  $a = 0.9$ . The critical radii are  $r_H = 0.718r_S$ ,  $r_L = 0.779r_S$ , and  $r_{\text{ms}} > r_S$ .

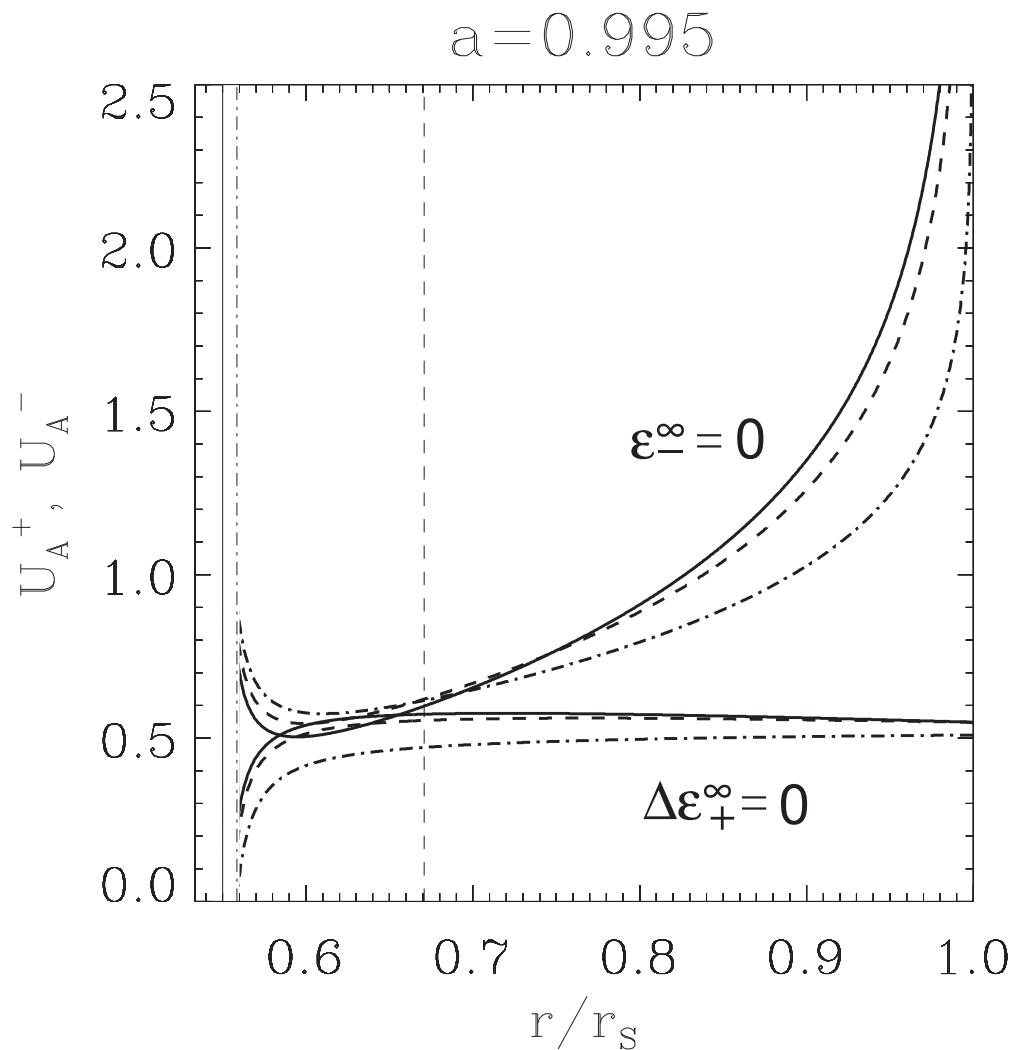


Fig. 7.— Similar to Fig. 5, but for the sub-Keplerian case  $c\hat{\beta}_0 = 0.6\hat{v}_K$ .

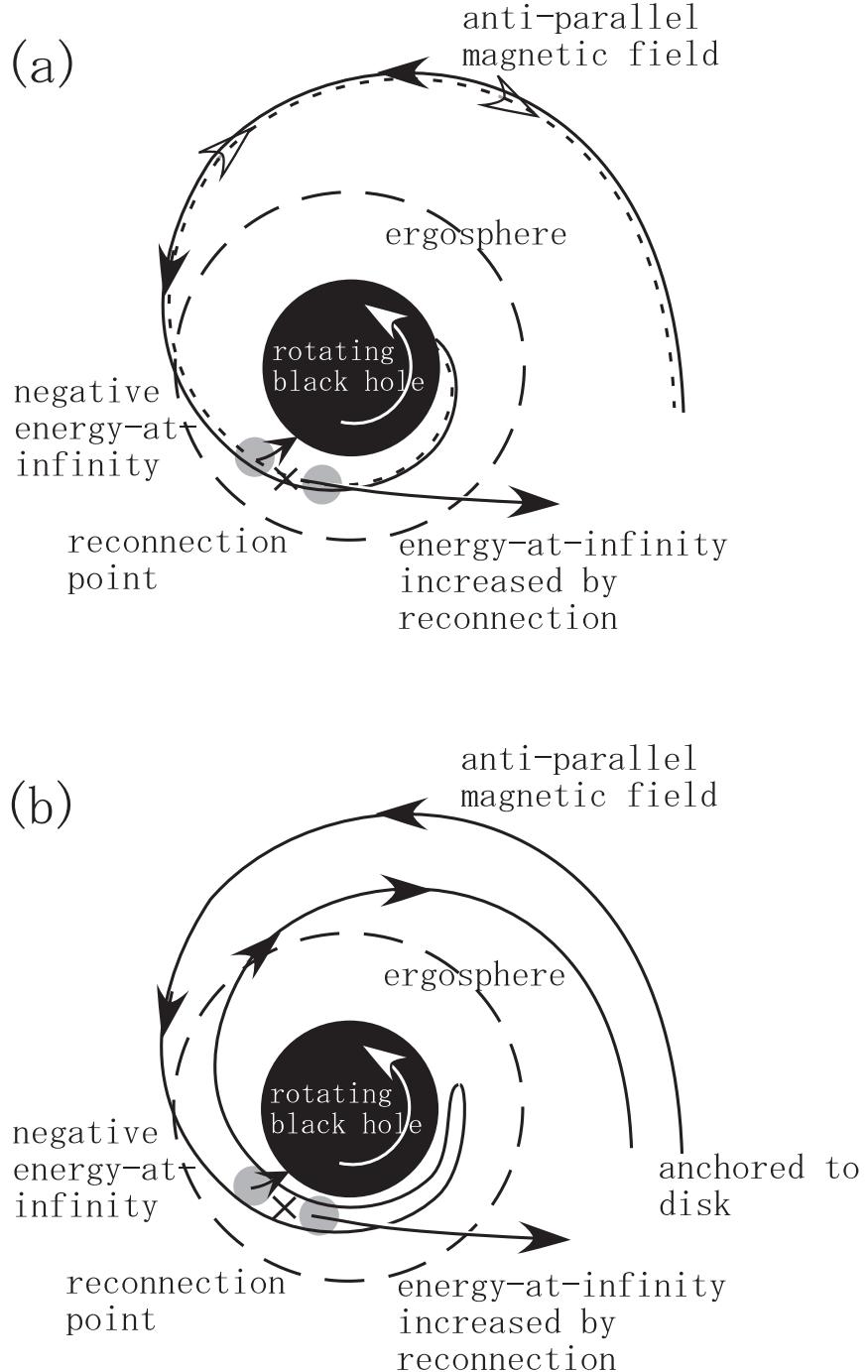


Fig. 8.— Astrophysical magnetic configuration of the magnetic reconnection in the black hole ergosphere. (a) Anti-parallel magnetic field caused from the initial uniform magnetic field around the rapidly rotating black hole. The solid/dashed line shows the anti-parallel magnetic field line in front of/behind the equatorial plane from the reader. In this magnetic field configuration, the current sheet locates at the equatorial plane. (b) Anti-parallel magnetic field formed by the closed magnetic flux tube tied to the disk around the rapidly rotating black hole. The inner part of the magnetic flux tube falls into the black hole to elongate the flux tube, and the anti-parallel magnetic field configuration is formed.